1. Transformations

a.) 2D affine transformation matrix $A$:

$$A [\tilde{a}_1 \tilde{a}_2 \tilde{a}_3] = [\tilde{b}_1 \tilde{b}_2 \tilde{b}_3],$$

where $\tilde{a}, \tilde{b}$ are column vectors representing 2D points of triangles in homogeneous coordinates.

$$A = [\tilde{b}_1 \tilde{b}_2 \tilde{b}_3] [\tilde{a}_1 \tilde{a}_2 \tilde{a}_3]^{-1}$$

$$A = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow 6 \text{ DoF} \rightarrow 6 \text{ unknowns}, 6 \text{ equations} \rightarrow 3 \text{ points mapped}$$

To be fully determined, $\tilde{a}$'s must be invertible $\rightarrow \tilde{a}$'s must be linearly independent $\rightarrow$ points must not be collinear.

b.) 2D homography $\rightarrow$ 8 DoF $\rightarrow$ 8 unknowns $\rightarrow$ 4 points mapped

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

2D similarity transform $\rightarrow$ 4 DoF $\rightarrow$ 2 points mapped

(x-translation, y-translation, uniform scale, rotation)

c.) yes, the centroid of a triangle is affine invariant.

no, the circumcenter of a triangle is not affine invariant.
(a) Because light travels in straight line through pinhole.

(b) 

\[
\begin{align*}
    w &= \frac{c - p}{||c - p||} \\
    v &= \frac{u \times w}{||u \times w||} \\
    a &= \frac{v \times u}{||v \times u||}
\end{align*}
\]

(c) Lines that are parallel to image plane

(d) Yes. They converge to a vanishing point. Each family of lines have a unique vanishing point. Even though that point might be outside of the screen.
\[ f(x, y, z) = (R - \sqrt{x^2 + y^2})^2 + z^2 - r^2 = 0, \quad R > r \]

\[ \frac{\partial f}{\partial x} = 2x + 2Rx \]
\[ \frac{\partial f}{\partial y} = 2y + 2Ry \]
\[ \frac{\partial f}{\partial z} = 2z \]
\[ \frac{\partial f}{\partial R} = \frac{x}{\sqrt{x^2 + y^2}} \]

\[ n_p = \left[ \begin{array}{c} \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{y}{\sqrt{x^2 + y^2}} \\ 0 \end{array} \right] \]
\[ t_p(x, y, z) = n_p \cdot \left( \frac{x}{r} - 1 \right) = 0 \]

\[ q(\lambda) = \begin{bmatrix} R \cos \lambda \\ R \sin \lambda \end{bmatrix} \]

\[ f(q(\lambda)) = (R - \sqrt{(R \cos \lambda)^2 + (R \sin \lambda)^2})^2 + r^2 = 0 \]
\[ = (R - \sqrt{R^2 \cos^2 \lambda + R^2 \sin^2 \lambda})^2 \]
\[ = (R - (R \cos \lambda + R \sin \lambda))^2 \]
\[ = (R - R)^2 = 0 \]

\[ \frac{\partial q}{\partial \lambda} = \begin{bmatrix} -R \sin \lambda \\ R \cos \lambda \\ 0 \end{bmatrix} \]

\[ \frac{\partial q}{\partial z} = \frac{\partial q}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial z} \]

(c) Sub \( q(\lambda) \) into \( n \) to get \( n_f \) then do \( n_f \cdot \frac{\partial q}{\partial \lambda} \).
4. \[ B_1(t) = (1-t)^3 P_1 + 3(1-t)^2 t P_2 + 3(1-t) t^2 P_3 + t^3 P_4 \]
\[ B_2(t-1) = (2-t)^3 P_4 + 3(2-t)^2 (t-1) P_5 + 3(2-t) (t-1)^2 P_6 + (t-1)^3 P_7 \]

\[ t \in [0,2] \]

a) \[ B'_1(1) = 3(P_4-P_3) \] \[ B'_2(1-1) = 3(P_5-P_4) \]

b) \[ B''_1(1) = 6(P_2-2P_3+P_4) \] \[ B''_2(1-1) = 6(P_6-2P_5+P_4) \]

c) For \( C^2 \) continuity, we need continuity up to and including 2nd derivative.

Since both curves share \( P_4 \), we know we have \( C^0 \) continuity.

For \( C^1 \):
\[ 3(P_4-P_3) = 3(P_5-P_4) \]

For \( C^2 \):
\[ 6(P_2-2P_3+P_4) = 6(P_6-2P_5+P_4) \]

\( P_1, P_2, P_3, P_4 \) are given for this question, so we solve in terms of \( P_1, P_2, P_3, P_4 \).

we get
\[ P_5 = 2P_4-P_3 \]
\[ P_6 = 2P_5-2P_3+P_2 = 4P_4-4P_3+2P_2 \]
\( P_7 \) is unconstrained

d) i) Affine Invariance
ii) Convex hull property
iii) Easy-to-compute derivatives
iv) Able to move out-of-plane

or others.

It's also good to write more than I did.

(Good luck on the final!)