Assignment 1 Part A. (I'm omitting a lot of explanation and showing my work. If you got the wrong answer, try working it out again yourself)

1. Implicit: \( \frac{y^2}{25} + \frac{x^4}{16} - \frac{x^2}{4} = 0 = f(x,y) \)

Tangent: \( \vec{t} = \left( 2 \cos(t), 5(\cos^2(t) - \sin^2(t)) \right) \quad \text{unit tangent: } \frac{\vec{t}}{\|\vec{t}\|} \)

Normal: \( \vec{n} = \nabla f(x,y) = \left( \frac{x^3}{4} - \frac{x}{2} \right) \quad \left( \frac{2y}{25} \right) = \left( 2 \sin^3(t) - \sin(t), 2 \sin(t) \cos(t) \right) \)

\[ \text{unit normal: } \frac{\vec{n}}{\|\vec{n}\|} \]

Symmetry: Symmetric in X and Y axes

Total Area: \( \frac{40}{3} \)

Piecewise Linear approximation: Discretize parameter \( t \) into \( n \) values between 0 and \( 2\pi \)

\[ t_i = \frac{2\pi i}{n} \quad i = 0, \ldots, n-1 \]

Then sample \( x(t) \) and \( y(t) \) at those points

\[ x_i = x(t_i) \]
\[ y_i = y(t_i) \]

Then, the length of the line between \( (x_i, y_i) \) to \( (x, y) \) is

\[ \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \]

We add these to approximate the perimeter

\[ P = \sum_{i=0}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \]

2. a) Yes, they commute. (Proof omitted)

b) No. Example: translation by \( (1,0) \) and rotation by \( 90^\circ \) (show your work)

C) No. Scaling by a factor of 2 on x, axis, 5 on y axis at fixed point \((1,1)\)
and rotation by \( 90^\circ \) at fixed point \((0,0)\)

d) Yes, they commute. (Proof omitted)
3. Transformation (Homography)

a) \[
\begin{bmatrix}
-1 & 0 & 2 \\
0 & 1 & 2 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
b) \[
\begin{bmatrix}
5 \\
2 \\
\end{bmatrix}
\]

4. Define 3 edge vectors

\( \vec{e}_0 = V_1 - V_0 \) with starting point \( V_0 \)
\( \vec{e}_1 = V_2 - V_1 \) with starting point \( V_1 \)
\( \vec{e}_2 = V_0 - V_2 \) with starting point \( V_2 \)

For each edge vector \( \vec{e} \) and starting point \( V \)

obtain the normal vector to \( \vec{e} \) by rotating by \(-90^\circ\) counterclockwise

\[ \vec{n} = \begin{bmatrix} 0 & -1 \end{bmatrix} \vec{e} \]

Get a point on the line represented by the edge. (\( \checkmark \))

Then our implicit form of this edge is

\[ \vec{n} \cdot (p - V) = 0 \]

Test \( \vec{n} \cdot (q - V) \)

If \( \vec{n} \cdot (q - V) = 0 \), \( q \) is on our edge.

If all edge-implicit tests have \( \vec{n} \cdot (q - V) \) being positive, or all
have \( \vec{n} \cdot (q - V) \) being negative, then \( q \) is inside. Otherwise \( q \) is outside.

(Note, the positive or negative testing depends on the orienting of the triangle (ie if \( V_0, V_1, V_2 \) are clockwise or counterclockwise).)

\[ V_0 \rightarrow V_1 \rightarrow V_2 \]